Vectorial Multi-machine Modeling for a Five-Phase Machine

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Abstract — To supply multi-phase machines and have a good control of torque it is necessary to control the currents with PWM VSI. In this case of supply, the steady-state models are not sufficient and we propose a new vectorial model of multi-phase synchronous machines. We prove that a multi-phase machine is equivalent to a group of 1-phase or 2-phase machines and show that is necessary to take into account more than the first harmonic of MMF. This paper describes the way of modeling and presents a new way of control.

1. Introduction

For industrial drives, the use of multi-phase synchronous machines with permanent magnets presents several advantages over conventional three-phase drives such as improved reliability, magnetic flux harmonic reduction, torque pulsation minimization and reduction of the power rating for the static converter [1] [8] [9]. The torque control imposes the use of current controllers associated with Pulse Width Modulation Voltage Source Inverter (PWM VSI). Steady-state models are not sufficient in case of supply by VSI and we propose a vectorial approach which uses a generalization of Concordia model, well known for three-phase machine. This model introduces the concept of equivalent machines, magnetically independent but electrically and mechanically coupled.

When synchronous machines with permanent magnets don’t possess sinusoidal magnetomotive force, it is necessary to take into account the harmonics of the MMF. Otherwise the harmonic currents generated by the PWM VSI are overvalued.

Thus, after modeling, the control of a multi-phase machine consists in controlling a set of one-phase or two-phase machines which can each one produce or not torque.

We present the model of a n-phase machine and we apply this method to a five-phase machine. We propose results with torque control by use of an original current control and compare it to classical one.

2. Modeling of a multi-phase machine

A. Assumptions and notations

Usual assumptions are used to model the machine:
- All phases are identical and regularly shifted by an angle

\[ \alpha = \frac{2\pi}{n} \]  \hspace{1cm} (1)

- Effects of saturation and amortisseur windings are neglected;
- Stator currents don’t modify the wave of the EMF induced in the stator coils by the rotor magnets.

All quantities relating to the phase \( k \) are written \( x_k \).

The n-phase machine is described in figure 1.

![Fig.1. Presentation of n-phase synchronous machine](image)

B. Modeling in a natural base

If an Euclidean vectorial space of dimension \( n \) is associated with the \( n \) phases, it is possible to define a vector with the \( n \) values of a quantity (for example the voltage) as coordinates.

An orthogonal base of this space is named natural and written \( B^r = \{x_k^r, x_2^r, ..., x_n^r\} \).

In this natural base, voltage and current vectors are defined:

\[ \vec{v} = v_1 \vec{x}^r_1 + v_2 \vec{x}^r_2 + ... + v_n \vec{x}^r_n \]  \hspace{1cm} (2)

\[ i = i_1 \vec{x}^r_1 + i_2 \vec{x}^r_2 + ... + i_n \vec{x}^r_n \]  \hspace{1cm} (3)
\[
\vec{v} = \vec{v} + \vec{v} + \ldots + \vec{v} + \vec{v}.
\]  
(3)

If a phase possesses a resistance \( R_x \), the vectorial voltage equation is:

\[
\vec{v} = R_x \vec{i} + \left[ \frac{d\vec{\phi}}{dt} \right]_{e_x} + \vec{e}
\]
(4)

and can be projected onto each vector of the natural base in order to find again the more classical equation:

\[
v_x = \vec{v} \cdot \vec{x}_x = R_x i_x + \frac{d\phi_x}{dt} + e_x
\]
(5)

where:
- \( \phi_x \) is the flux through the phase \( k \) exclusively created by the stator currents;
- \( e_x \) is the EMF induced in the phase \( k \) only due to the rotor magnets.

The assumptions previously defined allow to write a linear relation between the stator currents vector and the stator flux vector:

\[
\vec{\phi} = L \vec{i}
\]
(6)

usually written with the matrix notation:

\[
\begin{bmatrix}
L_{ix} & M_{ix} & \ldots & M_{ix} \\
M_{ix} & L_{ix} & \ldots & M_{ix} \\
\ldots & \ldots & \ldots & \ldots \\
M_{ix} & M_{ix} & \ldots & L_{ix}
\end{bmatrix}
\]

(7)

The modeling of the machine in the natural base is not adapted for the control of the machine because of the magnetic couplings between the phases. It is necessary to build a model where no magnetic couplings appear.

C. Modeling in a base which provides no magnetic couplings

The equation (6) is true whatever is the chosen base associated with the stator coils. The analysis of the inductance matrix (7) allows to affirm that:
- since the matrix is symmetric, it exists an orthogonal eigenvectors \( B' = \{x', x', \ldots, x'\} \), composed of the eigenvalues, within there is no magnetic couplings;
- if an eigenvalue \( L_x \) is multiple, there is a subspace \( E_x \) within the initial vectors can be projected onto. It is possible to find the vectorial equations in each subspace with a simple projection of the equations (5) and (7) onto a base of the chosen subspace.

Since the initial vectorial space can be divided into \( N \) orthogonal subspaces, the vector voltage \( \vec{v} \) is the sum of each vectorial projection \( \vec{v}_x \) of this vector onto each subspace:

\[
\vec{v} = \sum_{x=1}^{N} \vec{v}_x
\]
(8)

By a similar way there is a simple expression for the stator flux which shows the loss of the magnetic couplings:

\[
\vec{\phi} = \sum_{x=1}^{N} \vec{\phi}_x = \sum_{x=1}^{N} L_x \vec{i}_x
\]
(9)

For each subspace we can write the equation:

\[
\vec{v}_x = R_x \vec{i}_x + \left[ \frac{d\vec{\phi}_x}{dt} \right]_{e_x} + \vec{e}_x
\]
(10)

or with the use of equation (9) :

\[
\vec{v}_x = R_x \vec{i}_x + \left[ \frac{d\vec{i}_x}{dt} \right]_{e_x} + e_x
\]
(11)

D. Definition of multi-machine concept

The instantaneous power which passes through the real machine is:

\[
p = \sum_{x=1}^{N} \vec{v}_x \cdot \vec{i}_x = \vec{v} \cdot \vec{i}
\]
(12)

With the use of orthogonal subspaces previously defined, it is possible to write:

\[
p = \sum_{x=1}^{N} \vec{v}_x \cdot \vec{i}_x = \sum_{x=1}^{N} \vec{v}_x \cdot \vec{i}_x
\]
(13)

By injecting the equation (11) into this last equation, we obtain:

\[
p = \sum_{x=1}^{N} \left( R_x \frac{d\vec{i}_x}{dt} + L_x \vec{i}_x \right) \cdot \vec{i}_x + e_x \cdot \vec{i}_x
\]
(14)

Equation (14) shows that the power passes through \( N \) fictitious machines. Each fictitious machine has a number of phases equal to the multiplicity of the eigenvalue of the concerned subspace.

Thus, the initial machine can be projected onto each orthogonal subspace to form a set of fictitious machines. The whole group of fictitious machines introduces the concept of multi-machine.

The torque provided by the initial machine is equal to the sum of each torque provided by each fictitious machine:

\[
C = \sum_{x=1}^{N} C_x \text{ with } C_x = e_x \cdot \vec{i}_x.
\]
(15)

Every fictitious machines don’t produce torque but can generate parasite harmonic currents in case of little time constant and absence of EMF.

E. Classification and names of the fictitious machines

The magnitudes of each machine (voltage, current, EMF and flux) are obtained by projection of the vectors of the initial machine onto the subspaces previously defined.
It can be noticed that a group of harmonics is allocated to each fictitious machine.
We call the main machine the machine which can produce the greatest torque. In fact, this machine is associated with the subspace where the projections of the EMF are the greatest.
We call the homopolar machine the machine which is supplied by the homopolar magnitudes of the initial machine.
The other machines are respectively called: secondary, tertiary, etc...
The torque control of one multi-phase machine is become torque controls of several fictitious machines with no magnetic couplings but electrical and mechanical couplings.
The new system is now a multi-machine system.

3. Modeling of the voltage source inverter

A. Presentation of the initial supply

To increase reliability and to have more interchangeability, each phase of the machine is supplied by a single solid state [7]. There is no coupling between the phases and all switches can be separately controlled (There is a two-level control. Switches 1 and 4 have the same command and 2 and 3 to.) In this case there is no electrical couplings between the phases.
Figure 2 shows the structure of the initial VSI.

![Fig. 2. Structure of the initial VSI](image)

This VSI can supply the real machine by 2⁵ different voltage vectors and can generate any average voltage vector.

B. Supply of the fictitious machines

To know the voltage vectors which supply the fictitious machines, all we have to do is to project all the 2⁵ initial vectors onto each subspace. In this case, each machine is supplying by one fictitious multi-level inverter. The supply of one machine by a voltage vector imposes the supply by an other voltage vector to the other machines. Consequently, there is now an electrical coupling. The aim of the current control is to apply the right voltage vector on each machine in such a way to obtain the desired supply.
We can graphically represent these different couplings by using a formalism developed for the study of Multi-machine Multi-converter Systems (MMS) [2]

4. Modeling and torque control of a five-phase synchronous machine

A. Presentation of the machine and the VSI associated

The studied machine is a p-pole synchronous machine with permanent magnets and five phases. Each phase with pN turns is composed of concentrated windings.
In this study we respect the assumptions previously defined.
The electrical characteristics of the machine are resumed in the table 1.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>1500 W</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1500 rpm</td>
</tr>
<tr>
<td>Rated Bus voltage</td>
<td>150 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>5 A</td>
</tr>
<tr>
<td>Number of pairs of poles p</td>
<td>2</td>
</tr>
</tbody>
</table>

B. Modeling of the machine in the natural base, taking into account the MMF harmonics

a. Study of the inductance matrix

As shown in figure 4 and 5, the concentrated windings provide a non-sinusoidal MMF.

![Fig. 4. Representation of 2 concentrated windings of the five-phase machine](image)

![Fig. 5. Representation of the MMF](image)
The MMF acting across the air gap $g$ with the five stator currents is:

$$\zeta = K_1(\theta)x_1 + K_2(\theta)x_2 + \ldots + K_5(\theta)x_5$$  \hspace{1cm} (16)

The winding function for the winding $k$ is [10]:

$$K_k = \frac{2N}{\pi} \sum_{j=1}^{5} \frac{(-1)^{j+1}}{h^j} \cos \left( \hbar \left( \theta - (k-1) \frac{2\pi}{5} \right) \right), \ h = 2i + 1$$  \hspace{1cm} (17)

And with the assumptions previously defined, the field induction is:

$$B = \frac{\mu_0}{g} \times \zeta$$  \hspace{1cm} (18)

The flux through the winding $m$, being out of phase $\frac{2\pi}{5}p$ with the winding $k$, is given by:

$$\phi_m = \frac{2N}{\pi} \mu_0 D L \sum_{j=1}^{5} \frac{(-1)^{j+1}}{h^j} \cos \left( \hbar \left( k - m \right) \frac{2\pi}{5} \right) \times i_j$$  \hspace{1cm} (19)

with $D$ the average diameter and $L$ the length of the machine.

This relation between all the flux and currents allows to write the vectorial equation (6) where the component of the inductance matrix is:

$$L_{mk} = \frac{2N}{\pi g} \mu_0 D L \sum_{j=1}^{5} \frac{(-1)^{j+1}}{h^j} \cos \left( \hbar \left( k - m \right) \frac{2\pi}{5} \right)$$  \hspace{1cm} (20)

In a classic approach with the first harmonic, only the fundamental harmonic of $L_{mk}$ appears. It can be noticed that there are only odd harmonics in the harmonic inductances.

b. Study of the EMF

The EMF induced at 1500 rpm into a winding is shown in figure 6.

![Fig 6. Representation of the EMF](image)

In table 2, a Fourier analysis shows the amplitude of all harmonics.

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Value as a percentage of the fundamental</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>28.5</td>
</tr>
<tr>
<td>5</td>
<td>12.4</td>
</tr>
<tr>
<td>7</td>
<td>5.1</td>
</tr>
<tr>
<td>9</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Other harmonics will be neglected.

C. Multi-machine modeling

a. Classification of sets of harmonics

The multi-machine concept consists in projecting all magnitudes vectors onto a sub-space where it exists no magnetic couplings between the phases (Generalization of Concordia’s transformation).

The initial machine can be decomposed onto three fictitious machines such as:
- One two-phase machine called main;
- One two-phase machine called secondary;
- One one-phase machine called homopolar.

The linear transformation which allows to find the magnitudes of the fictitious machines shows that a set of odd harmonics ($h=1, 3, 5, 7, \ldots$) is projected onto each machine.

These sets are:
- harmonics 1, 9, 11, ..., $5h \pm 4, \ldots$ for the main machine;
- harmonics 3, 7, ..., $5h \pm 2, \ldots$ for the secondary machine;
- harmonics 5, 15, ..., $5h, \ldots$ for the homopolar machine.

b. Characteristics of the fictitious machines

The two-phase main machine possesses an inductance:

$$L_r = \frac{5}{2} \sum \frac{L_{h}}{h^2} + L_{\text{total}}$$  \hspace{1cm} (21)

and an EMF composed with the harmonics $h = 1, 9, \ldots$ of the initial EMF.

The two-phase secondary machine possesses an inductance:

$$L_{\text{end}} = \frac{5}{2} \sum \frac{L_{h}}{h^2} + L_{\text{total}}$$  \hspace{1cm} (22)

and an EMF composed with the harmonics $h = 3, 7, \ldots$ of the initial EMF.

The one-phase homopolar machine possesses an inductance:

$$L_{\text{sub}} = \frac{5}{2} \sum \frac{L_{h}}{h^2} + L_{\text{total}}$$  \hspace{1cm} (23)

and an EMF composed with the harmonics $h = 5, 15, \ldots$ of the initial EMF.

Figure 7 shows the EMFs of the three machines.
It can be noticed that the main machine has the greatest time constant $L/R$, and EMF. This is the machine which can produce, for given current and frequency of PWM, the highest torque with the least high harmonic currents. The secondary machine can produce less torque than the main one and possesses a smaller time constant. The currents of this machine will be contain, for a same frequency of PWM, more high harmonics than the currents in the main machine. The homopolar machine has the smallest time constant and can’t produce a constant torque due the fact it is a one-phase machine.

c. Fictitious VSI

The different vectors which supply the fictitious machines are obtained by projecting the $2^5$ initial vectors onto the two planes (respectively associated to the main and the secondary machine) and onto a line (associated to the homopolar machine) [5] [6] [12]. Figures 8, 9 and 10 show their space vector representations.

Fig 9. Space Vector Representation of the VSI which supplies the secondary machine

D. Torque control of the machine

The control of the five-phase machine is a now a torque control of three machines. The figure 11 shows the structure of the current control [11]. To have the highest constant torque with a simple design of control system, we choose to supply only the main and the secondary machines. The only harmonics of EMF used to create torque are the fundamental of the main machine and the third-harmonic of the secondary machine.

This structure provides constant references of currents by a rotation transformation and uses Proportional Integral controllers instead of hysteresis controllers. As the PWM frequency is constant, the spectrum of current is less wide than with classic hysteresis controllers. Figure 12 shows the currents into the phases of the three fictitious machines. It can be notice that the high frequency harmonic currents are higher in the secondary and the homopolar compared to the main one. It is due to the difference between the time constants of the machines. Figure 14 shows the torque produced by the main and the secondary machines. The sum of these torques is the torque produced by the five-phase machine.
It is possible to obtain a little more torque, using the other harmonics, but than, the complexity of the control increases.

and many fictitious machines magnetically independent. A multi-phase machine can be considered like a set of one or two-phase machines. The complex torque control of a multi-phase machine is become a more simple and robust control of many two-phase or one-phase machines. With the use of PWM for the current control, the spectrum of the current phases is better controlled and limited than with classic hysteresis current control.

An analysis of the MMF generated by the coils shows that there is more than the leakage inductance in the secondary and homopolar machines. A study with Finite Element Method is planed to calculate the harmonic inductances [3] [4]. To limit the high harmonic currents, it is interesting to consider that windings with more MMF harmonics increase the time constant of the secondary and homopolar machines. Thus, the frequency of the PWM can be reduced by the use of windings with no sinusoidal MMF.

This analysis allows the study of the current control structures as well as the design of the statoric windings and the rotoric magnets.

5. Conclusion

In this paper we have shown that a vectorial analysis provides a powerful way to model multi-phase machines. This analysis allows to generalize the transformation of Concordia and can be applied whatever is the number of phases of the machine. Moreover, this analysis introduces the concept of equivalence between a multi-phase machine

References


