Optimum use of DC bus by fitting the back-electromotive force of a 7-phase Permanent Magnet Synchronous machine

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Abstract
This paper deals with design constraints of a 7-phase Permanent Magnet Synchronous Machine (PMSM) supplied by a 7-leg Voltage Source Inverter. The optimum back electromotive force waveform is determined in order to get maximum torque for a given DC bus voltage.

Introduction
Multi-phase DC brushless machines suffer from an apparent higher number of switching devices than 3-phase ones. Nevertheless, in high power applications such as electrical ship [1]-[2] or low voltage/high current applications such as on-board systems (traction) [3]-[5], this factor is not so obvious: use of high current devices implies high heat dissipation capabilities especially with high frequencies. Then it is common to use either parallel converters or parallel/serie device associations.

Moreover, when reliability is required such as in aircraft [6], in marine applications [7]-[8] and in offshore variable speed wind generators, multi-phase drives [10]-[11] must be considered as an alternative to 3-phase multi-level converter drives whose reconfiguration in safety mode is not obvious.

In this paper, an axial double-rotor 7-phase virtual prototype is considered and has been modelled with 3D-finite element method. The global aim is to fit the machine to its 7-leg Voltage Source Inverter (VSI) in order to optimize the global drive.

For 3-phase machines supplied by 3-leg VSI, the optimum use of the DC bus voltage has been widely studied [12]. It consists in injecting a third harmonic component in the voltage references of the VSI when triangle intersection method is employed or to use a space-vector modulation [13]. When the 3-phase machine is wye-coupled the injection of a third harmonic component has impact neither on the torque nor on the currents of the machine. For wye-coupled 7-phase machines supplied by 7-leg VSI (Fig. 1), the problem is quite all different. The injection of a third component implies currents and eventually torque components in the machine. A Multi-Machine modelling is used in the paper to prove and explain this difference. Nevertheless the injection of a third harmonic component remains interesting for an optimal use of the 7-leg VSI DC bus Voltage [14].
The aim of this paper is to find necessary fitting of the machine in order to be able to inject a maximum third harmonic component in the reference voltages. At first, a Multi-Machine modelling of a 7-phase axial permanent magnet machine is presented: it allows to transform a complex problem into simpler ones. Then, for a given machine and a maximum value of the first harmonic voltage components, effects of injection of a third harmonic voltage component are studied: it appears that results depend on the harmonic spectrum of the back-electromotive force. Finally, thanks to the Multi-Machine modelling, machine design constraints are deduced in order to take maximum advantage of a third harmonic voltage component: extra torque is produced for a given DC bus voltage.

Multi-Machine vectorial characterization

Under assumptions of no saturation, no reluctance effects and regularity of design, a vectorial formalism allows to prove that a 7-phase machine is equivalent to a set of three magnetically independent fictitious 2-phase machines [15] named $M1$, $M2$ and $M3$. Each equivalent machine is characterized by its inductance (resp. $L_{M1}$, $L_{M2}$ and $L_{M3}$), resistance (resp. $R_{M1}$, $R_{M2}$ and $R_{M3}$), and back-EMF (resp. $e_{M1}$, $e_{M2}$ and $e_{M3}$). The torque of the real machine $T$, is the sum of the torque of these three machines $T_{M1}$, $T_{M2}$ and $T_{M3}$. The 7-leg VSI can also be decomposed into three fictitious VSI electrically coupled by a mathematical transformation Concordia’s type [15]. A fictitious VSI is characterized by a set of space phasors as it is the case for 3-leg VSI (with the usual hexagonal representation).

The equivalence is based on a generalized Concordia transformation characterized by the $[C_7]$ matrix:

$$[C_7] = \frac{2}{\sqrt{7}} \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\frac{1}{\sqrt{2}} & \cos \frac{2\pi}{7} & \sin \frac{2\pi}{7} & \cos \frac{4\pi}{7} & \sin \frac{4\pi}{7} & \cos \frac{6\pi}{7} & \sin \frac{6\pi}{7} \\
\frac{1}{\sqrt{2}} & \cos \frac{4\pi}{7} & \sin \frac{4\pi}{7} & \cos \frac{8\pi}{7} & \sin \frac{8\pi}{7} & \cos \frac{12\pi}{7} & \sin \frac{12\pi}{7} \\
\frac{1}{\sqrt{2}} & \cos \frac{6\pi}{7} & \sin \frac{6\pi}{7} & \cos \frac{12\pi}{7} & \sin \frac{12\pi}{7} & \cos \frac{18\pi}{7} & \sin \frac{18\pi}{7} \\
\frac{1}{\sqrt{2}} & \cos \frac{8\pi}{7} & \sin \frac{8\pi}{7} & \cos \frac{16\pi}{7} & \sin \frac{16\pi}{7} & \cos \frac{24\pi}{7} & \sin \frac{24\pi}{7} \\
\frac{1}{\sqrt{2}} & \cos \frac{10\pi}{7} & \sin \frac{10\pi}{7} & \cos \frac{20\pi}{7} & \sin \frac{20\pi}{7} & \cos \frac{30\pi}{7} & \sin \frac{30\pi}{7} \\
\frac{1}{\sqrt{2}} & \cos \frac{12\pi}{7} & \sin \frac{12\pi}{7} & \cos \frac{24\pi}{7} & \sin \frac{24\pi}{7} & \cos \frac{36\pi}{7} & \sin \frac{36\pi}{7}
\end{bmatrix} \tag{1}
$$

Relationships between values of fictitious machines and real values (noted with subscripts 1, 2, ..., 7) are then defined by:

$$i_{mach} = \begin{bmatrix}
i_{M1a} & i_{M1\beta} & i_{M2a} & i_{M2\beta} & i_{M3a} & i_{M3\beta}
\end{bmatrix}^T = [C_7]^T \begin{bmatrix}
i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7
\end{bmatrix} \tag{2}
$$

$$v_{mach} = \begin{bmatrix}
v_{M1a} & v_{M1\beta} & v_{M2a} & v_{M2\beta} & v_{M3a} & v_{M3\beta}
\end{bmatrix}^T = [C_7]^T \begin{bmatrix}
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7
\end{bmatrix} \tag{3}$$

Fig. 1: symbolic representation of 7-leg PWM-VSI and wye-coupled 7-phase machine
Currents and voltages obtained using this transformation can be decomposed into three subsystems associated with the $M1$, $M2$ and $M3$ machines:

$$
\begin{align*}
\vec{v}_{M1} &= \begin{bmatrix} v_{M1a} & v_{M1\beta} \end{bmatrix} \\
\vec{v}_{M2} &= \begin{bmatrix} v_{M2a} & v_{M2\beta} \end{bmatrix} \\
\vec{v}_{M3} &= \begin{bmatrix} v_{M3a} & v_{M3\beta} \end{bmatrix} \\
\vec{i}_{M1} &= \begin{bmatrix} i_{M1a} & i_{M1\beta} \end{bmatrix} \\
\vec{i}_{M2} &= \begin{bmatrix} i_{M2a} & i_{M2\beta} \end{bmatrix} \\
\vec{i}_{M3} &= \begin{bmatrix} i_{M3a} & i_{M3\beta} \end{bmatrix}
\end{align*}
$$

(A4)

A key of the problem is that each one of the 2-phase fictitious machine is characterized by an harmonic family (Table I) and a vectorial subspace $S_k$. The three subspaces are orthogonal each to other. It is this orthogonality which allows to introduce the concept of fictitious machine.

### Table 1: Harmonic characterization of fictitious machines for wye-coupled 7-phase machine

<table>
<thead>
<tr>
<th>Fictitious 2-phase machines</th>
<th>Families of odd harmonics</th>
</tr>
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<tbody>
<tr>
<td>$M1$</td>
<td>$1, 13, 15, \ldots, 7h \pm 1$</td>
</tr>
<tr>
<td>$M2$</td>
<td>$5, 9, 19, \ldots, 7h \pm 2$</td>
</tr>
<tr>
<td>$M3$</td>
<td>$3, 11, 17, \ldots, 7h \pm 3$</td>
</tr>
</tbody>
</table>

To get a synthetic representation, a graphical formalism (Energetic Macroscopic Representation: EMR) is used (see Appendix and Fig. 2). Interleaved triangles traduce a mechanical coupling between the three fictitious machines ($T = T_{M1} + T_{M2} + T_{M3}$). Interleaved squares traduce an electrical coupling: the three fictitious VSI are supplied by only one DC bus.

The voltage equations of these $M1$ and $M3$ machines are:

$$
\begin{align*}
\vec{v}_{M1} &= R_{M1}\vec{i}_{M1} + L_{M1} \frac{di_{M1}}{dt} + \vec{e}_{M1} \\
\vec{v}_{M3} &= R_{M3}\vec{i}_{M3} + L_{M3} \frac{di_{M3}}{dt} + \vec{e}_{M3}
\end{align*}
$$

(A5)

The electromechanical conversion is traduced by equation (6):

$$
\vec{e}_{M1} \cdot \vec{i}_{M1} = T_{M1} \Omega \\
\vec{e}_{M2} \cdot \vec{i}_{M2} = T_{M2} \Omega \\
\vec{e}_{M3} \cdot \vec{i}_{M3} = T_{M3} \Omega
$$

(A6)

Fig. 2: Multi-Machine Energetic Macroscopic Representation of the 7-phase machine
We get finally from (22) and (23), \( e_M = e_{M1} + e_{M3} \), the total back-EMF of the real machine. Its projection relatively to the phase n°1 gives the back-EMF represented in Fig. 9.

**Determination of extra torque**

The extra torque \( T_{M3} \) developed by the \( M3 \) machine is 25N.m, value which represents 20% of the nominal torque (125N.m) developed by the \( M1 \) machine. Of course, extra Joule losses appears. Nevertheless, the imposed constraint of minimum losses (\( i_M = k e_M \)) allows to reduce their increase to 20%. If we impose to work with the same Joule losses, by reducing \( i_{M1} \) and \( i_{M3} \) in the same ratio, the torque increase is 9%.

**Conclusions**

It has been shown that, as for 3-phase machines, it was possible to take advantage of injection of a third harmonic component of voltage. The DC bus voltage is then better used, as it was with the 3-phase machines. Nevertheless, for optimum use of the DC bus voltage, the 7-phase machine must be fitted to the 7-leg VSI. With improvements of design of PM rotor, such as Halbach arrays, such optimal design can be considered. Of course, the obtained extra torque is a “booster” torque for transient states, unless improved heat dissipation is achieved.
References


Appendix: Synoptic of Energetic Macroscopic Representation